AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MD02

Unit Decision 2

Tuesday 24 June 2014   9.00 am to 10.30 am

For this paper you must have:
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed
• 1 hour 30 minutes

Instructions
• Use black ink or black ball-point pen. Pencil should only be used for drawing.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
• You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
• Do not write outside the box around each page.
• Show all necessary working; otherwise marks for method may be lost.
• Do all rough work in this book. Cross through any work that you do not want to be marked.
• The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 75.

Advice
• You do not necessarily need to use all the space provided.

MD02

Answer **all** questions.

Answer each question in the space provided for that question.

1 A major project has been divided into a number of tasks, as shown in the table.
The minimum time required to complete each task is also shown.

Activity	Immediate predecessor	Duration (hours)
A	–	3
B	A	3
C	A	4
D	B, C	6
E	B, C	5
F	C	2
G	C	1
H	A	15
I	D, E	4
J	F	6
K	G	10
L	H, I, J, K	1

(a) On the page opposite, construct an activity network for the project.
(Activity A has already been drawn.)

[3 marks]

(b) Find the earliest start time for each activity.

[2 marks]

(c) Find the latest finish time for each activity.

[2 marks]

(d) List the critical activities.

[2 marks]

QUESTION PART REFERENCE **Answer space for question 1**

Answer space for question 1

Earliest start time

Duration

Latest finish time

A
3

Turn over ▶

QUESTION PART REFERENCE	Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

Turn over ▶

2 Alex and Roberto play a zero-sum game. The game is represented by the following pay-off matrix for Alex.

Roberto

	Strategy	D	E	F	G
	A	5	−4	−1	1
Alex	**B**	4	3	0	1
	C	−3	0	−5	−2

(a) Show that this game has a stable solution and state the play-safe strategy for each player.

[4 marks]

(b) List any saddle points.

[1 mark]

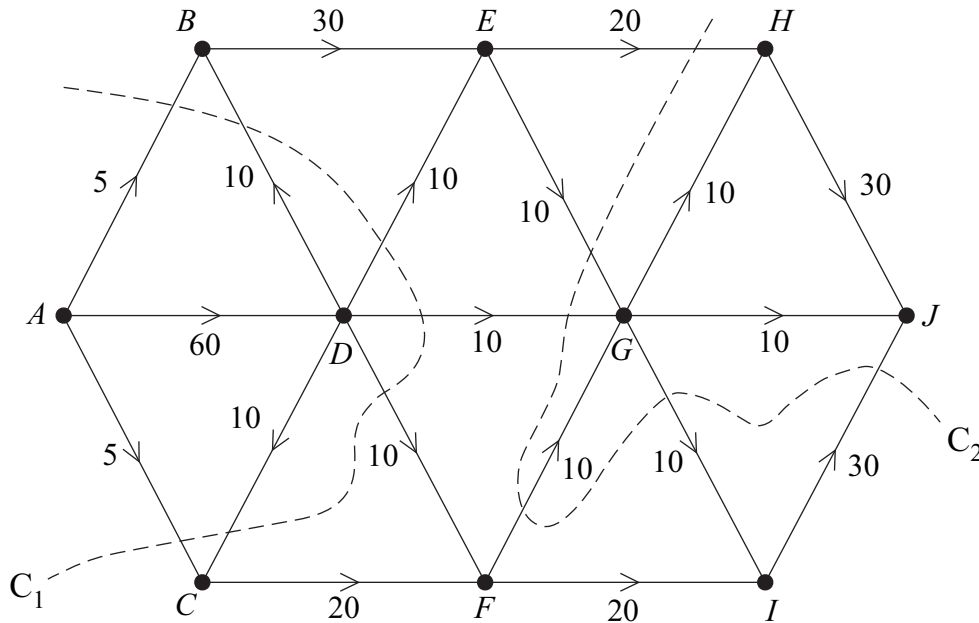| QUESTION PART REFERENCE	**Answer space for question 2**
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QUESTION PART REFERENCE	Answer space for question 2

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3 The diagram below shows a network of pipes with source A and sink J. The capacity of each pipe is given by the number on each edge.

(a) Find the values of the cuts C_1 and C_2.

[2 marks]

(b) Find by inspection a flow of 60 units, with flows of 25, 10 and 25 along HJ, GJ and IJ respectively. Illustrate your answer on **Figure 1**.

[2 marks]

(c) (i) On a certain day the section EH is blocked, as shown on **Figure 2**.

Find, by inspection or otherwise, the maximum flow on this day and illustrate your answer on **Figure 2**.

[3 marks]

(ii) Show that the flow obtained in part **(c)(i)** is maximal.

[2 marks]

Answer space for question 3

QUESTION PART REFERENCE	**Answer space for question 3**
(b)	
(c)	

Figure 1

B   E   H

25

A   D   G   J

10

25

C   F   I

Figure 2

B   E   H

A   D   G   J

C   F   I
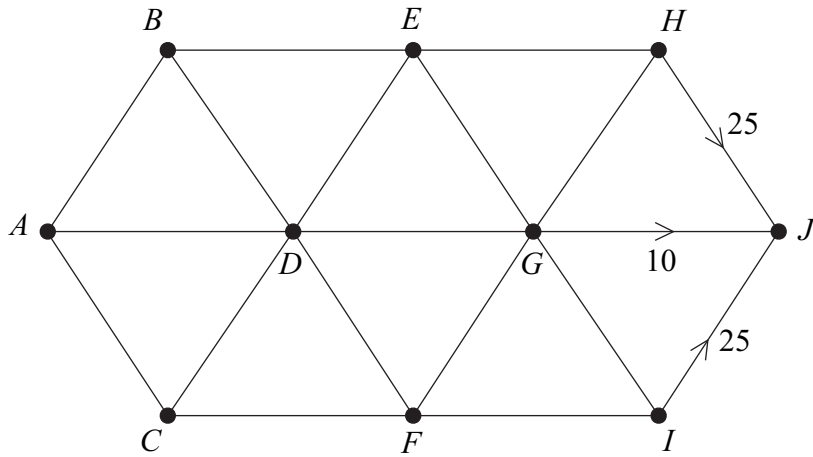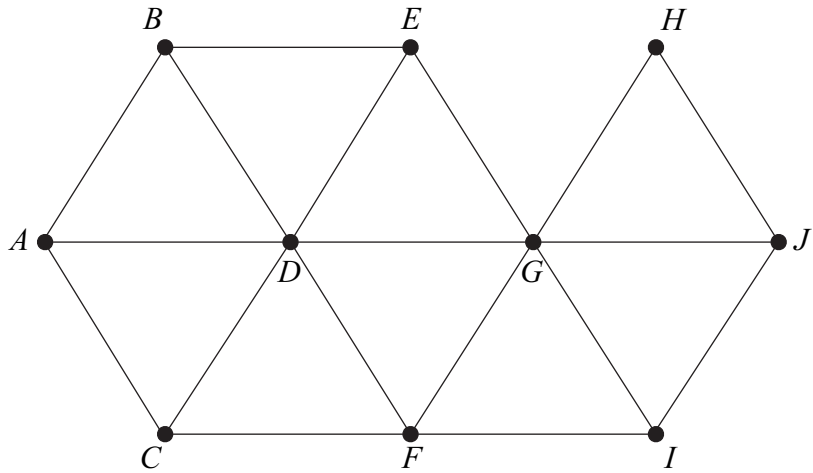
Maximum flow = _____

Turn over ▶

QUESTION PART REFERENCE	Answer space for question 3

QUESTION PART REFERENCE	Answer space for question 3

4 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 3x + 6y + 2z$

subject to $x + 3y + 2z \leqslant 11$

$3x + 4y + 2z \leqslant 21$

and $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$.

[2 marks]

(b) The first pivot to be chosen is from the y-column.

Perform one iteration of the Simplex method.

[3 marks]

(c) Perform one further iteration.

[3 marks]

(d) Interpret the tableau obtained in part **(c)** and state the values of your slack variables.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

| QUESTION
PART
REFERENCE	Answer space for question 4

Turn over ▶

5 Mark and Owen play a zero-sum game. The game is represented by the following pay-off matrix for Mark.

Owen

Strategy	D	E	F
A	4	1	−1
B	3	−2	−2
C	−2	0	3

Mark

(a) Explain why Mark should never play strategy B.

[1 mark]

(b) It is given that the value of the game is 0.6. Find the optimal strategy for **Owen**.

(You are **not** required to find the optimal mixed strategy for Mark.)

[7 marks]

QUESTION PART REFERENCE	Answer space for question 5

QUESTION PART REFERENCE	Answer space for question 5

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6 The network below has 11 vertices and 16 edges connecting some pairs of vertices. The numbers on the edges are their weights. The weight of the edge DG is given in terms of x.

There are three routes from A to K that have the same minimum total weight.
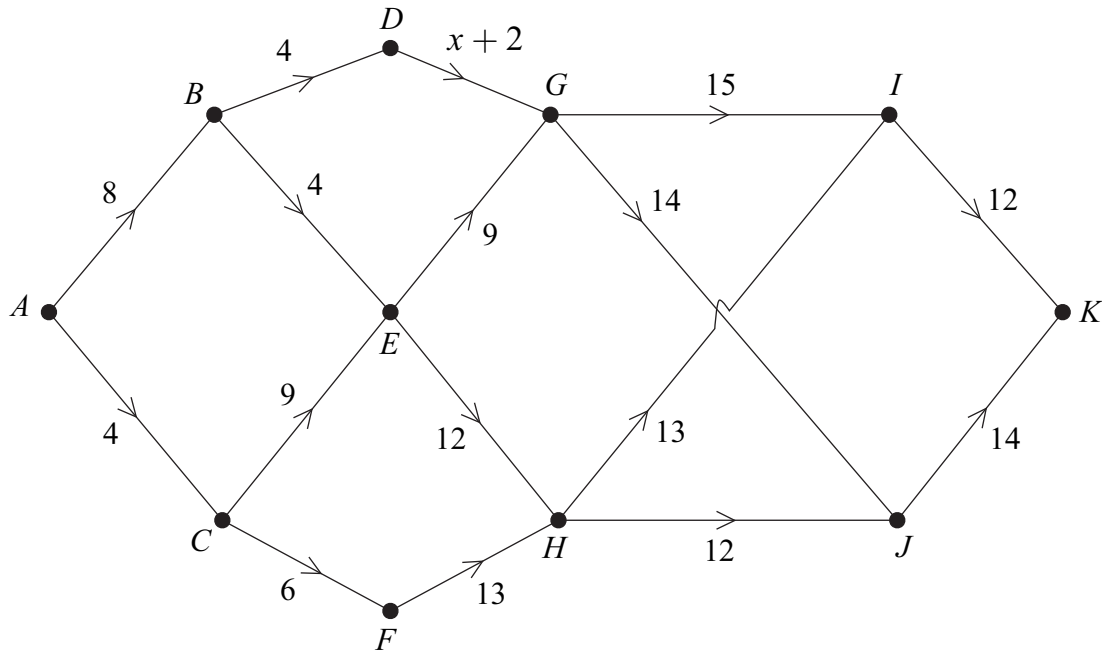
Working backwards from K, use dynamic programming, to find:

(a) the minimum total weight from A to K ;

(b) the value of x ;

(c) the three routes corresponding to the minimum total weight.

You must complete the table opposite as your solution.

[12 marks]

QUESTION PART REFERENCE	Answer space for question 6

Answer space for question 6

Stage	State	From	Calculation	Value
1	I	K		
	J	K		

QUESTION PART REFERENCE	Answer space for question 6

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QUESTION PART REFERENCE	Answer space for question 6

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7 The table shows the times taken, in minutes, by four people, A, B, C and D, to carry out the tasks W, X, Y and Z.

Some of the times are subject to the same delay of x minutes, where $4 < x < 11$.

	A	B	C	D
Task W	$x + 8$	$x + 4$	$x + 6$	$x + 9$
Task X	$x + 5$	$x + 3$	$x + 4$	$x + 2$
Task Y	$x + 8$	$x + 7$	$x + 5$	$2x + 2$
Task Z	$x + 3$	$2x - 3$	12	$x + 1$

Each of the four tasks is to be given to a different one of the four people so that the total time for the four tasks is minimised.

(a) The minimum time to complete task Z is $(x + 1)$.

Write down the minimum time to complete task W, task X and task Y.

[2 marks]

(b) Use the Hungarian algorithm, by reducing the **rows** first, to assign each task to a different person so that the total time for the four tasks is minimised.

[7 marks]

(c) Given that the minimum total time is 42 minutes, find the value of x.

[2 marks]

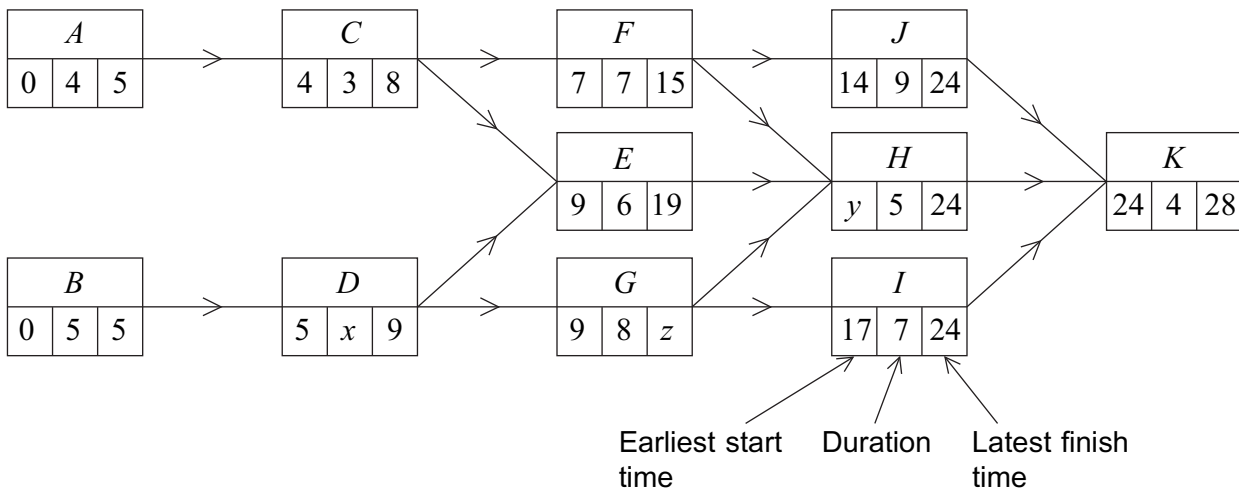| QUESTION PART REFERENCE	**Answer space for question 7**
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QUESTION PART REFERENCE	Answer space for question 7

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8 An activity diagram for a project is shown below. The duration of each activity is given in weeks. The earliest start time and the latest finish time for each activity are shown on the diagram.

	A	
0	4	5

	C	
4	3	8

	F	
7	7	15

	J	
14	9	24

	E	
9	6	19

	H	
y	5	24

	K	
24	4	28

	B	
0	5	5

	D	
5	x	9

	G	
9	8	z

	I	
17	7	24

Earliest start time   Duration   Latest finish time

(a) Find the values of x, y and z.

[2 marks]

(b) State the critical path.

[1 mark]

(c) Some of the activities can be speeded up at an additional cost. The following table lists the activities that can be speeded up together with the minimum possible duration of these activities. The table also shows the additional cost of reducing the duration of each of these activities by one week.

Activity	Additional cost per week (£)	Minimum completion time (weeks)
E	8000	1
F	7000	4
G	6000	5

The company wishes to complete the project as soon as possible.

(i) Find which activities should be speeded up. For **each** such activity, state, with justification, the reduction in the number of weeks.

(ii) Hence state the revised minimum time for the completion of the whole project.

(iii) Calculate the total additional cost that the company would incur in meeting this revised completion time.

[7 marks]

QUESTION PART REFERENCE	Answer space for question 8

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END OF QUESTIONS